Composite Functions

1. For the following pairs of functions, find $fg(x)$.

   a. $f(x) = 3x + 5$ and $g(x) = 2x$

   b. $f(x) = x - 1$ and $g(x) = 3x + 2$

   c. $f(x) = 2x + 1$ and $g(x) = 3 - x$

   d. $f(x) = x^2$ and $g(x) = x + 2$

   e. $f(x) = 3x^2 - x$ and $g(x) = 2x - 3$

2. The function f is such that $f(x) = 3x - 1$.

   Find $ff(3)$.

3. The function f is such that $f(x) = 2 - 4x$ and $g(x) = 3x$. Solve the equation $gf(x) = 18$.

4. The functions f and g are such that $f(x) = 2x$ and $g(x) = 3x + 8$.

   a. Find $fg(x)$.

   b. Hence solve the equation $fg(a) = 4a$.

5. The functions f and g are such that $f(x) = x^2 + 3$ and $g(x) = \frac{2}{x}$.

   a. Find $fg(x)$.

   b. Find $gf(x)$.

6. The functions f and g are such that $f(x) = x + c$ and $g(x) = 2x - 1$.

   Given that, for a constant a, $fg(a) = 10$ and $gf(a) = 15$, find the value of the constants a and c.

7. The function g is such that $g(x) = 2x^2 + 1$.

   Find $gg(2)$.

8. The functions f, g and h are such that $f(x) = 3x$, $g(x) = 2x - 3$ and $h(x) = 5x$.

   Find $fgh(x)$.

9. The functions f and g are such that $f(x) = \frac{x}{(x+5)}$ and $g(x) = 2x + 1$. Find $fg(x)$.

Composite Functions **Answers**

1. a. $6x + 5$

   b. $3x + 1$

   c. $7 - 2x$

   d. $(x + 2)^2 = x^2 + 4x + 4$

   e. $12x^2 - 38x + 30$

2. $9x - 4$

3. $x = -1$

4. a. $6x + 16$

   b. -8

5. a. $\frac{4}{x^2} + 3$

   b. $\frac{2}{(x^2 + 3)}$

6. $a = 3, c = 5$

7. $gg(x) = 8x^4 + 8x^2 + 3$

   so $gg(2) = 50$.

8. $30x - 1$

9. $\frac{2x + 1}{2x + 6}$

Algebra

Composite Functions

Learning Objective

To find composite functions.

Success Criteria

- To understand and interpret composite function notation.
- To find composite functions using two functions.
- To apply these skills to GCSE style questions.

Starter

Find the answer to each question.

Put your answers in terms of size, from smallest to largest, to spell out a keyword.

Can you give a definition for this word?

1. The function **d** is such that $d(x) = 4x + 2$. Find the value of $d(-3)$ **−10**
2. The function **m** is such that $m(x) = 2x - 3$. Find the value of $m(1.5)$ **0**
3. The function **a** is such that $a(x) = x^2$. Find the value of $a(\frac{1}{4})$ $\frac{1}{16}$
4. The function **o** is such that $o(x) = 3x + 4$. Find the value of $o^{-1}(1)$ **−1**
5. The function **n** is such that $n(x) = \frac{2}{3}x - 7$. Find the value of $n^{-1}(5)$ **18**
6. The function **i** is such that $i(x) = 2x^2$. Find the (positive) value of $i^{-1}(32)$ **4**

domain - the set of x values that are the input of the function.

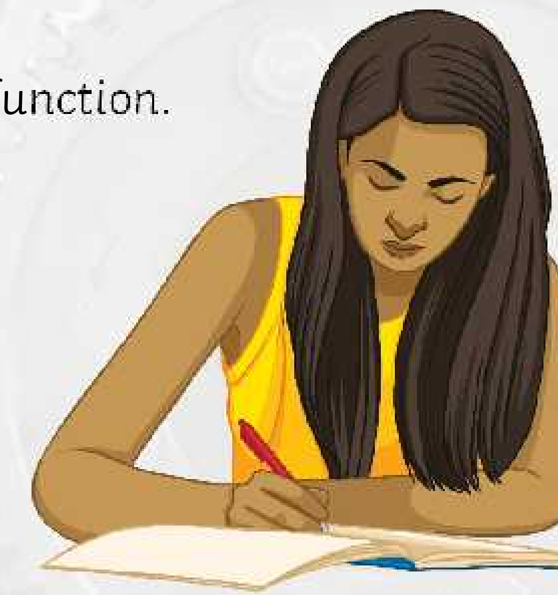
Composite Functions

A composite function is a function consisting of at least two functions.
It is written as $fg(x)$ or $gf(x)$ depending on the order.

To help you work out what order to do the functions in, try rewriting it.

For example: $fg(x) = f(g(x))$

So you apply the g function first, then the f function.



Worked Example

The functions f and g are defined as $f(x) = 2x + 1$ and $g(x) = 3x$. Find $fg(x)$.

Step one: Rewrite your composite function.

$fg(x) = f(g(x))$

Step two: Where you see $g(x)$ appear in the brackets, substitute in $3x$, since we are told that $g(x) = 3x$.

$f(g(x)) = f(3x)$

Step three: Remember, if you see $f(7)$, you substitute 7 every time you see x. Here, we see $f(3x)$, so we must substitute $3x$ every time we see x.

$f(3x) = 2(3x) + 1$

So $fg(x) = 6x + 1$

Worked Example

The functions f and g are defined as $f(x) = 2x + 1$ and $g(x) = 3x$. Find $gf(x)$.

Step one: Rewrite your composite function.

$$gf(x) = g(f(x))$$

Step two: Where you see $f(x)$ appear in the brackets, substitute in $2x + 1$ since we are told that $f(x) = 2x + 1$.

$$g(f(x)) = g(2x + 1)$$

Step three: Every time you see x, substitute in $2x + 1$.

$$g(2x + 1) = 3(2x + 1)$$

So $gf(x) = 6x + 3$

Think, Pair, Share

The functions f and g are defined as $f(x) = 4x - 2$ and $g(x) = x + 5$. Find $fg(x)$.

Step one: Rewrite your composite function.

$fg(x) = f(g(x))$

Step two: Where you see $g(x)$ appear in the brackets, substitute in $x + 5$ since we are told that $g(x) = x + 5$.

$f(g(x)) = f(x + 5)$

Step three: Every time you see x, substitute in $x + 5$.

$f(x + 5) = 4(x + 5) - 2$

So $fg(x) = 4x + 18$

Think, Pair, Share

The functions f and g are defined as $f(x) = x + 4$ and $g(x) = x^2$. Find $gf(x)$.

Step one: Rewrite your composite function.

$gf(x) = g(f(x))$

Step two: Where you see $f(x)$ appear in the brackets, substitute in $x + 4$ since we are told that $f(x) = x + 4$.

$g(f(x)) = g(x + 4)$

Step three: Every time you see x, substitute in $x + 4$.

$g(x + 4) = (x + 4)^2$

So $gf(1) = (1 + 4)^2 = 25$

Your Turn

Complete the **Activity Sheet**.

Extension:

A function f is defined as
$f(x) = 4(3x - 7)$.

1. Find $f^{-1}(x)$.

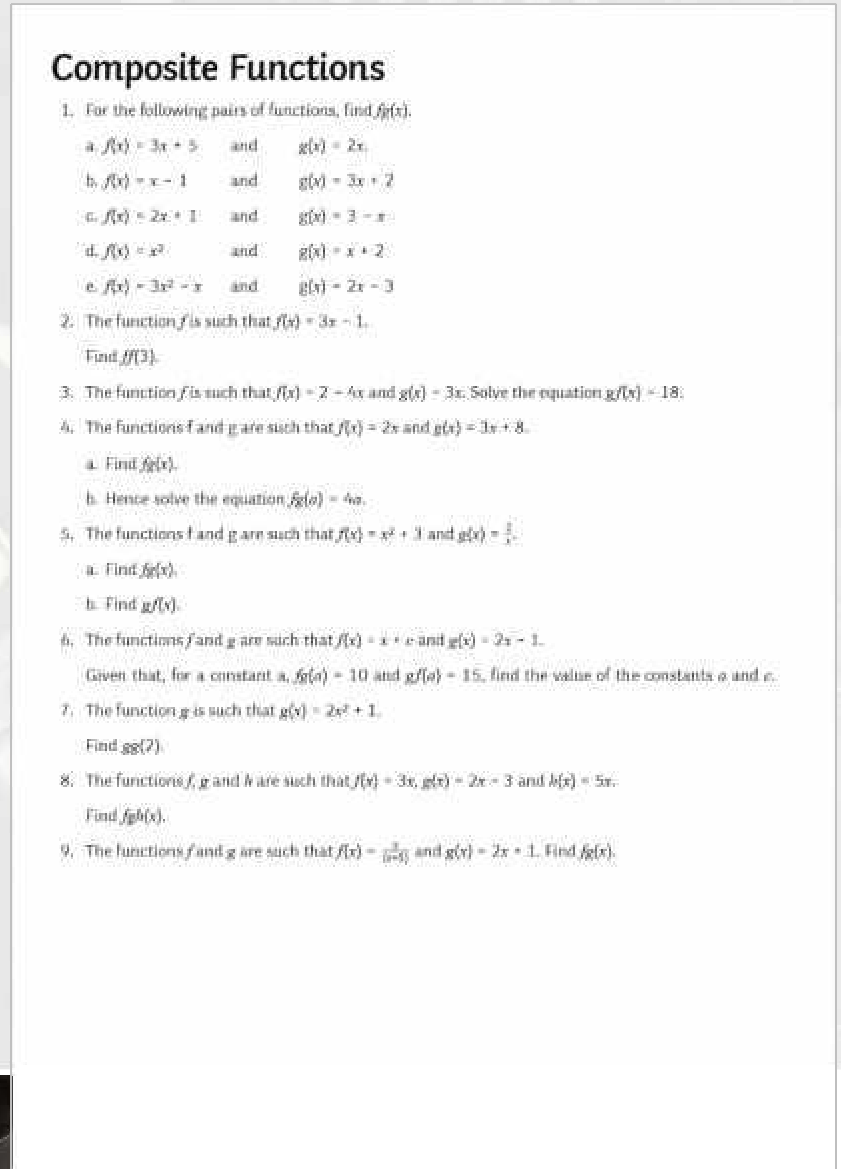
   $f^{-1}(x) = \frac{x + 28}{12}$

2. Find $ff^{-1}(x)$ and $f^{-1}f(x)$.

   $ff^{-1}(x) = x$ **and** $f^{-1}f(x) = x$

   What do you notice?
   Why do you think this is?

   This is because f and f^{-1} are inverse functions so one always reverses the other.



Composite Functions

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   Find $gg(2)$.
8. The functions f, g and h are such that $f(x) = 3x$, $g(x) = 2x - 3$ and $h(x) = 5x$.
   Find $fgh(x)$.
9. The functions f and g are such that $f(x) = \frac{3}{(x+5)}$ and $g(x) = 2x + 1$. Find $fg(x)$.

Plenary

The functions f and g are such that $f(x) = 3x - 2$ and $g(x) = kx^2$.

Given that $fg(-2) = 34$, find the value of k.

$$fg(x) = 3kx^2 - 2$$

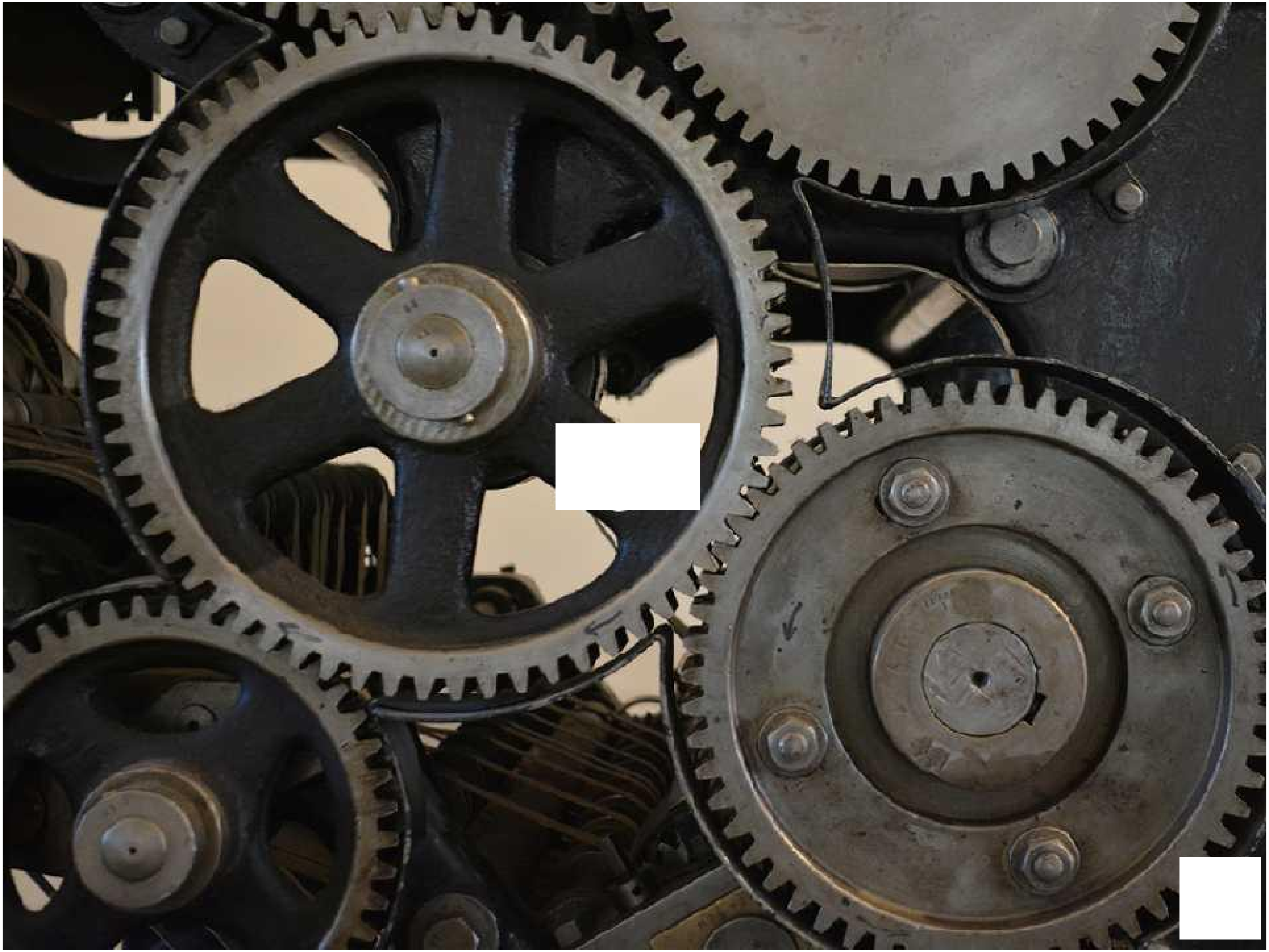
$$fg(-2) = 3 \times k \times (-2)^2 - 2$$

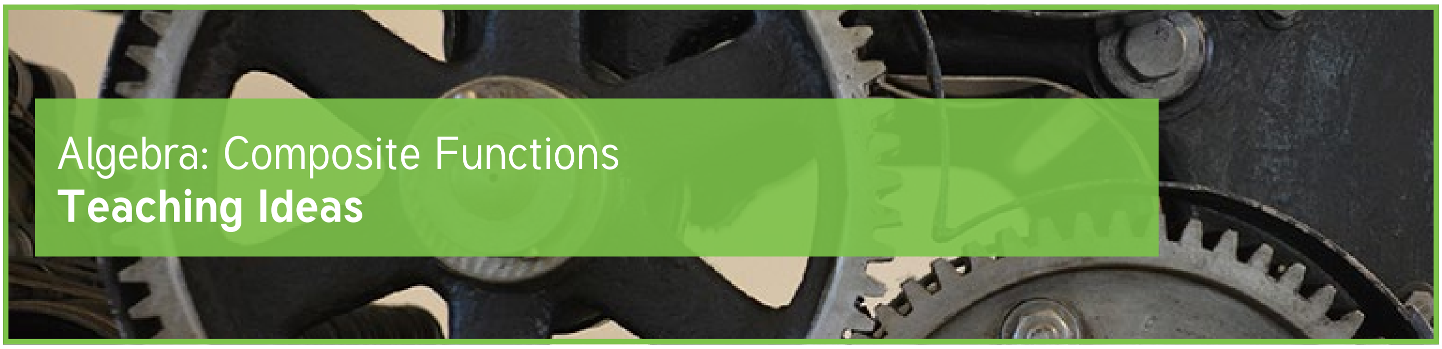
$$fg(-2) = 12k - 2$$

$$12k - 2 = 34$$

$$12k = 36$$

$$k = 3$$





Algebra: Composite Functions
Teaching Ideas

Learning Objective: To find composite functions.

Success Criteria:

- To understand and interpret composite function notation.
- To find composite functions using two functions.
- To apply these skills to GCSE style questions.

Context: This is a standalone lesson, but should be completed after students have a solid understanding on function notation and finding inverse functions.

Starter

Students complete the questions given, then order their answers from smallest to largest to spell out the word domain. Whilst it is not expected that they know the meaning of this word for their GCSE, it is a simple definition for discussion.

Main Activities

Composite Functions

Show students the correct notation, emphasising the value in adding brackets to the function to make it clear which order you must apply them. Go through the worked examples as a class, encouraging students to make detailed notes.

Think, Pair, Share

Students work in pairs to check their understanding before discussing any misconceptions as a class.

Your Turn

Students work independently to complete the **Composite Functions Activity Sheet**. Extension and answers provided. Be prepared to support less able students though the notation $fg(a) = 4a$ and the need to develop and solve simultaneous equations in question 6.

Plenary

Students should work through the exam question in pairs. A natural extension is to discuss where they think the marks might come from in an exam (this would likely be a 4-mark question, with 2 marks coming from finding $fg(x)$, 1 mark from the correct substitution for $fg(-2)$ and the final mark for solving correctly to find the value of k).